國立高雄科技大學

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Hough Transform:
A Case Study of Line Detection

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Outline

- Introduction
  - History
  - Purpose
  - Idea
- Detection Algorithm
  - Shape Parameterization
  - Parameter Voting

Introduction

- History
  - It is an algorithm patented by **P. Hough** in 1962.
  - It was used for lines and simple polygons in 1972.

    R. Duda and P. Hart, “Use of the Hough Transformation to Detect Lines and Curves ”, Commun ACM , 1972

  - It was generalized to arbitrary shapes in 1981.

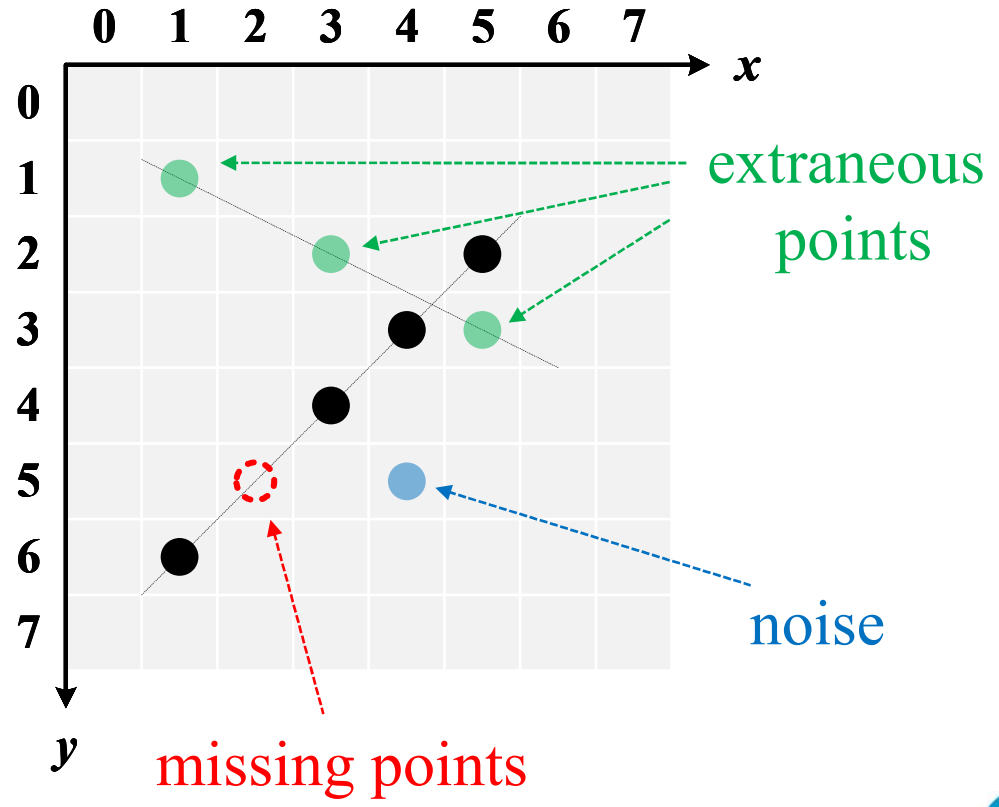
    Ballard DH, “Generalizing the Hough Transform to Detect Arbitrary Shapes,” Pattern Recognition, 1981.

Introduction

- Purpose
  - detect the shape from the boundary points but in presence of **imperfections**.
    - missing points: some boundary points are not detected from edge detector
    - extraneous points: some detected boundary points may come from other objects
    - noise: some non-boundary points may also be detected as boundary points.

Introduction

- Purpose

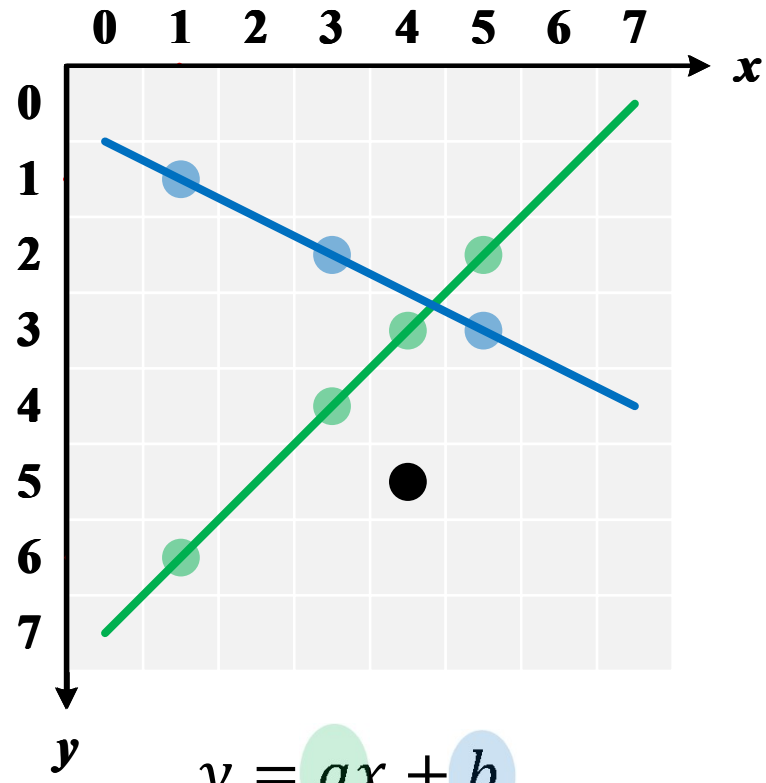


Introduction

- Idea
  - Global Approach: use **all** boundary points for shape detection ⇨ permit missing boundary points
  - Voting Scheme
    - count **occurrence number** of the parameter values.
    - take the parameter values with **higher votes** as the estimation results.

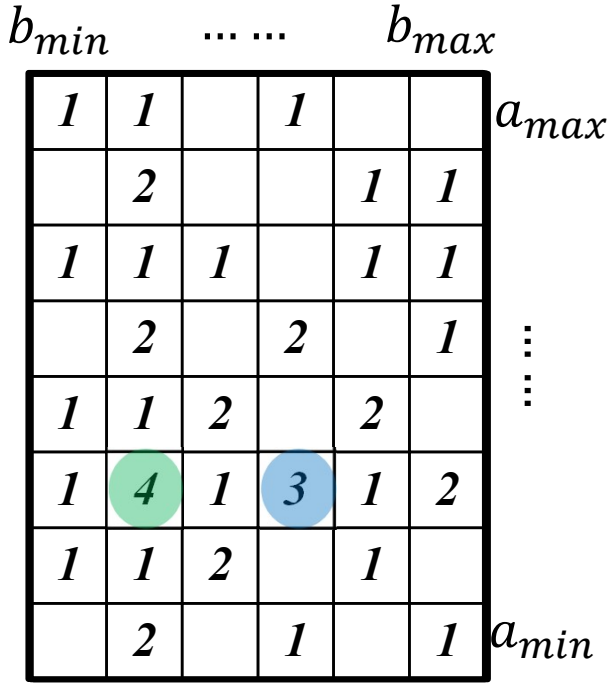
    ⇨ alleviate the effect from extraneous and noisy points

Introduction



$$y = ax + b$$

slope intercept



parameter accumulator

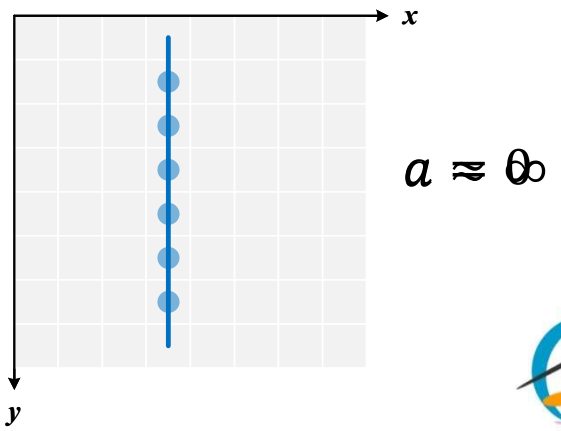
Detection Algorithm

- Shape Parameterization
  - formulate the shape detection as **parameter estimation** problem.

$$y = ax + b \implies (a, b) \text{ estimation}$$

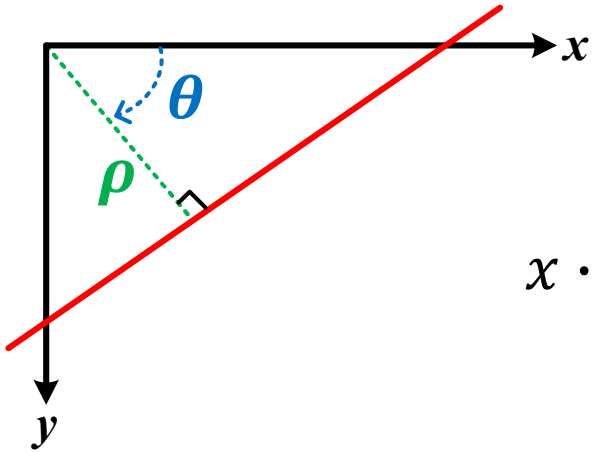
  - condition: all parameters must have **finite** ranges.

slope parameter a
has infinite range



Detection Algorithm

- Shape Parameterization: Line Detection
  - use (ρ, θ) instead of (a, b) parameter space
    - ρ: the distance between the line and the origin
    - θ: the angle of the vector from the origin to the norm



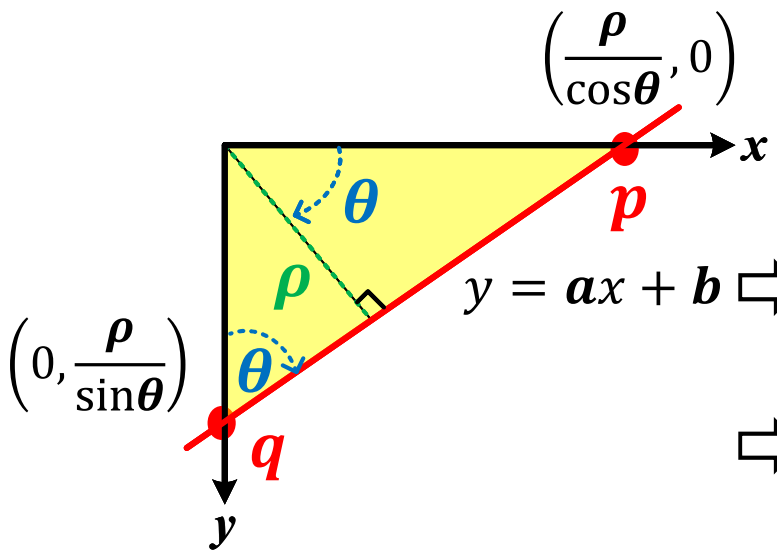
$$y = ax + b$$

$$\Downarrow$$

$$x \cdot \cos\boldsymbol{\theta} + y \cdot \sin\boldsymbol{\theta} = \boldsymbol{\rho}$$

Detection Algorithm

- Shape Parameterization: Line Detection
  - use (ρ, θ) instead of (a, b) parameter space



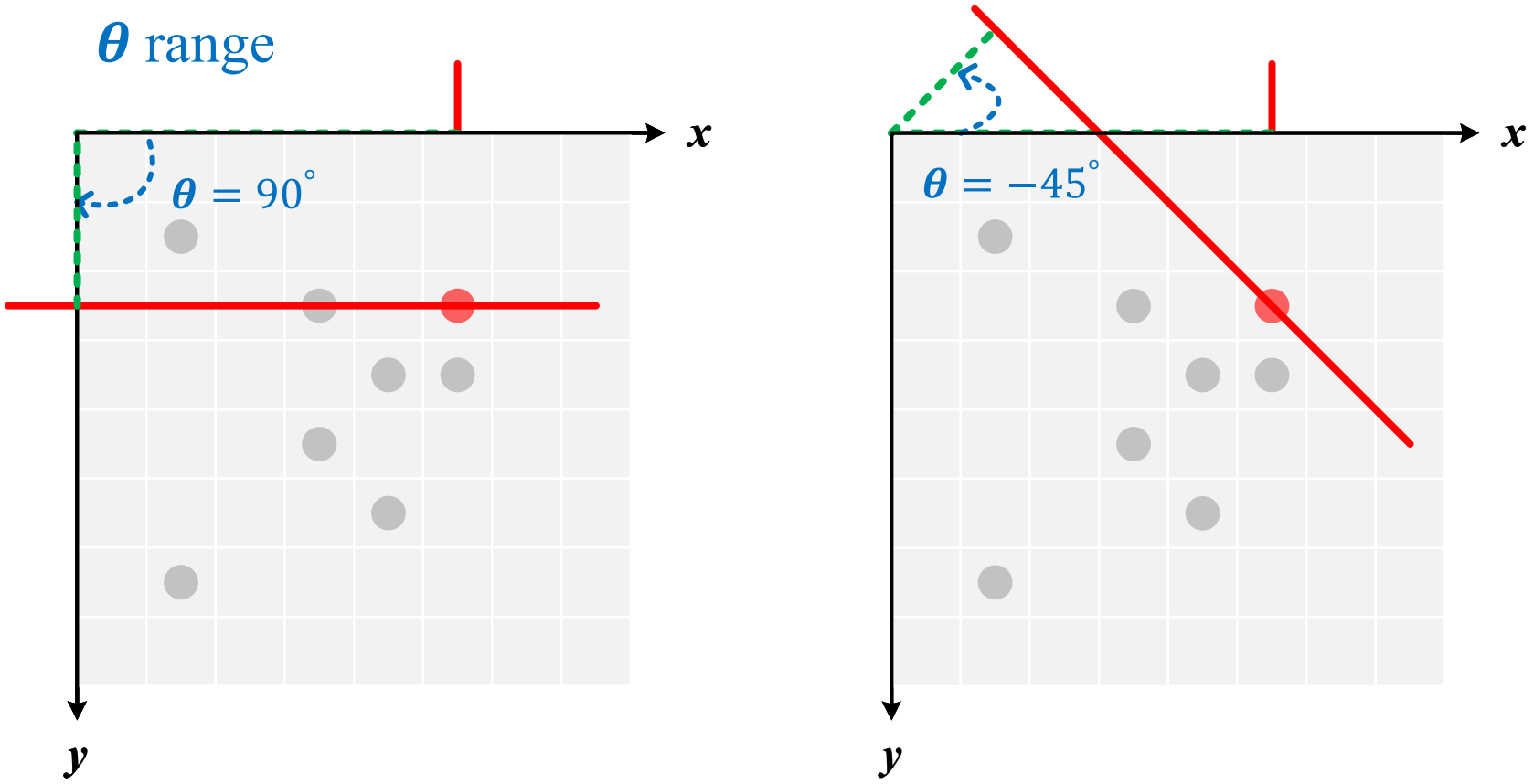
$$a = \frac{\left(\frac{\rho}{\sin\theta} - 0\right)}{\left(0 - \frac{\rho}{\cos\theta}\right)} = -\frac{\cos\theta}{\sin\theta} \qquad b = \frac{\rho}{\sin\theta}$$

$$y = ax + b \Rightarrow y = -\frac{\cos\theta}{\sin\theta} \cdot x + \frac{\rho}{\sin\theta}$$

$$\Rightarrow y \cdot \sin\theta = -\cos\theta \cdot x + \rho$$

$$\Rightarrow x \cdot \cos\theta + y \cdot \sin\theta = \rho$$

Detection Algorithm

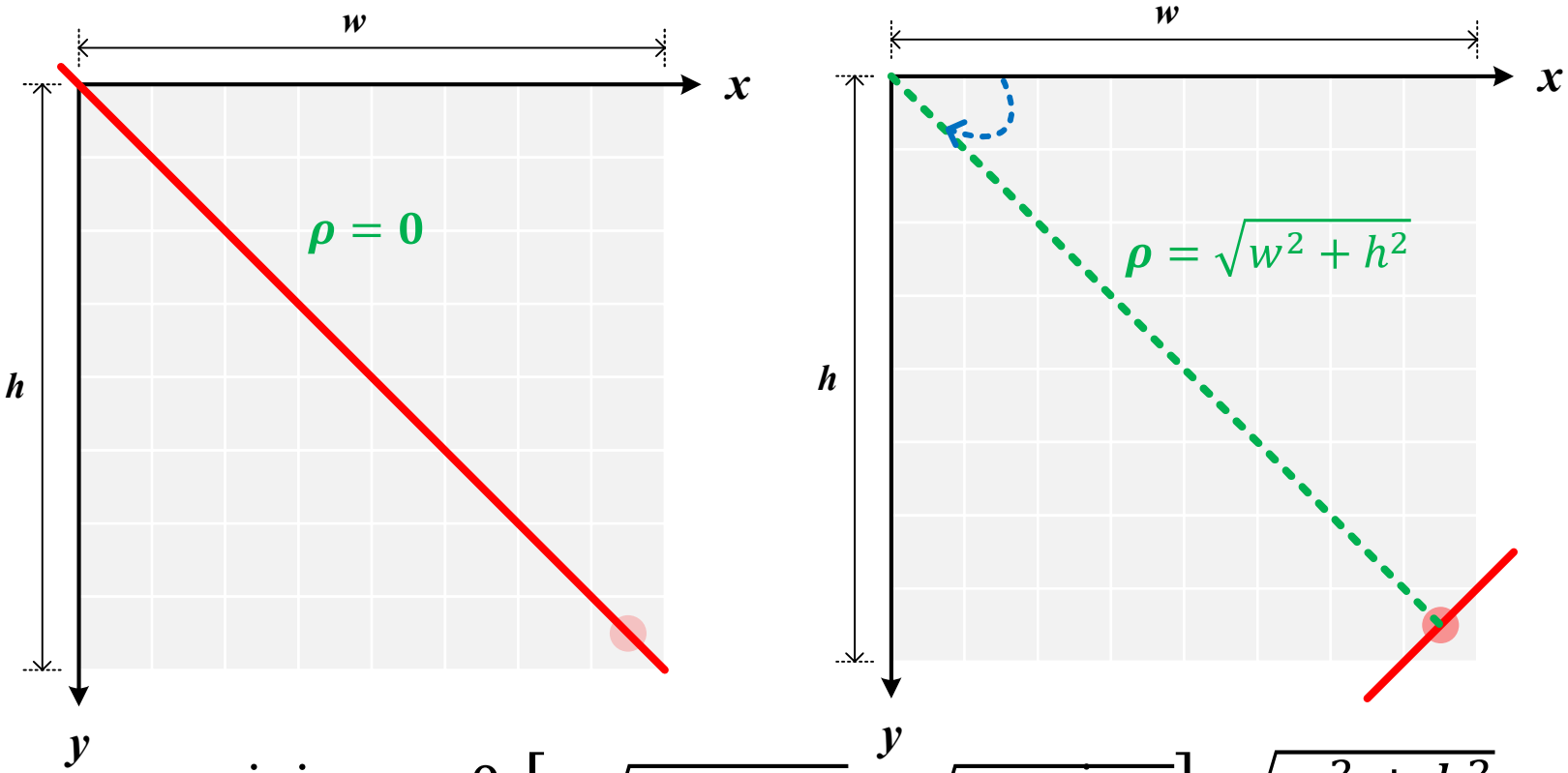


clockwise: $0^\circ \rightarrow 90^\circ$ counterclockwise: $0^\circ \rightarrow -90^\circ$

θ range: $[-90^\circ, +90^\circ]$

Detection Algorithm

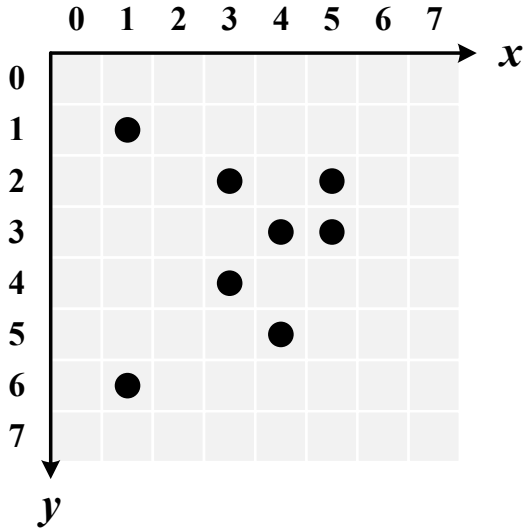
ρ range

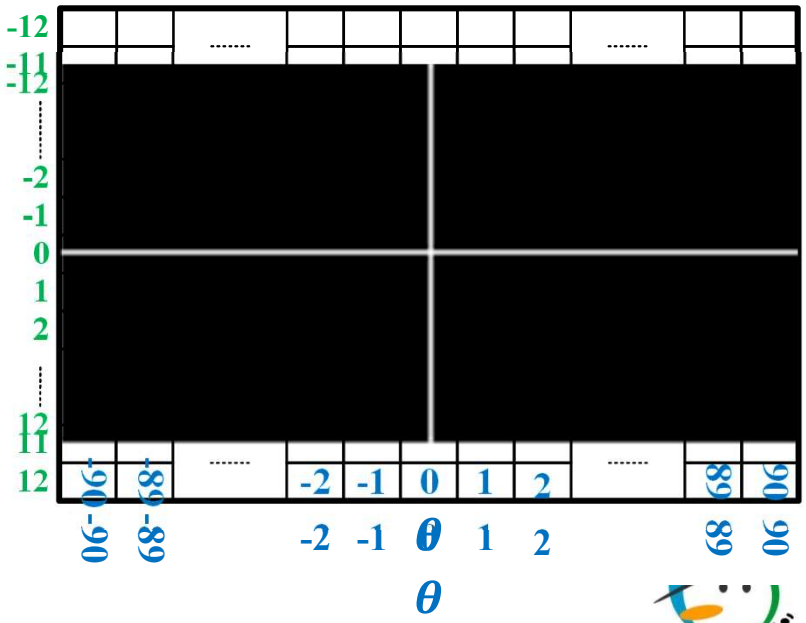


minimum: 0 ρ range: $\left[-\sqrt{w^2+h^2}, +\sqrt{w^2+h^2}\right]$ maximum: $\sqrt{w^2+h^2}$

Detection Algorithm

- Parameter Voting
  - quantize parameter space into several bins
  - create a parameter accumulator $A(.)$





ρ range: $\left[-\sqrt{8^2+8^2}, +\sqrt{8^2+8^2}\right]$

Detection Algorithm

- Parameter Voting
  - take all boundary point $p(x, y)$ for voting all possible parameters.
    - for θ_i from -90 to $+90$
      - compute the ρ_i

        $$x \cdot \cos\boldsymbol{\theta_i} + y \cdot \sin\boldsymbol{\theta_i} = \boldsymbol{\rho_i}$$

      - increase the bins $A(\boldsymbol{\rho_i}, \boldsymbol{\theta_i})$ by one

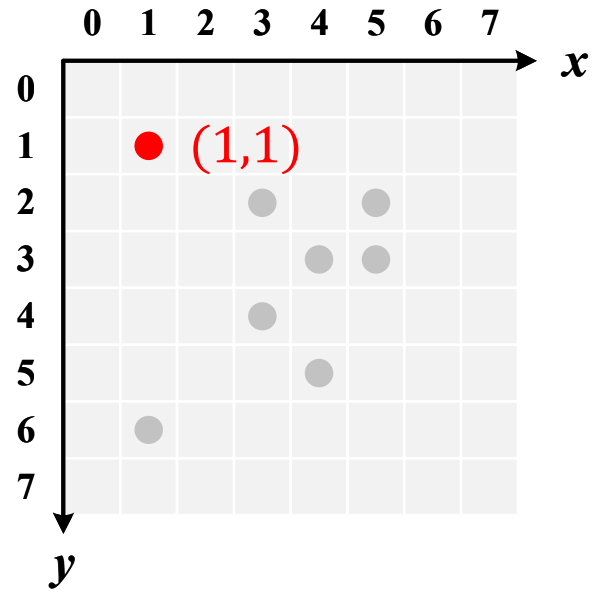
        $$A(\boldsymbol{\rho_i}, \boldsymbol{\theta_i}) = A(\boldsymbol{\rho_i}, \boldsymbol{\theta_i}) + 1$$

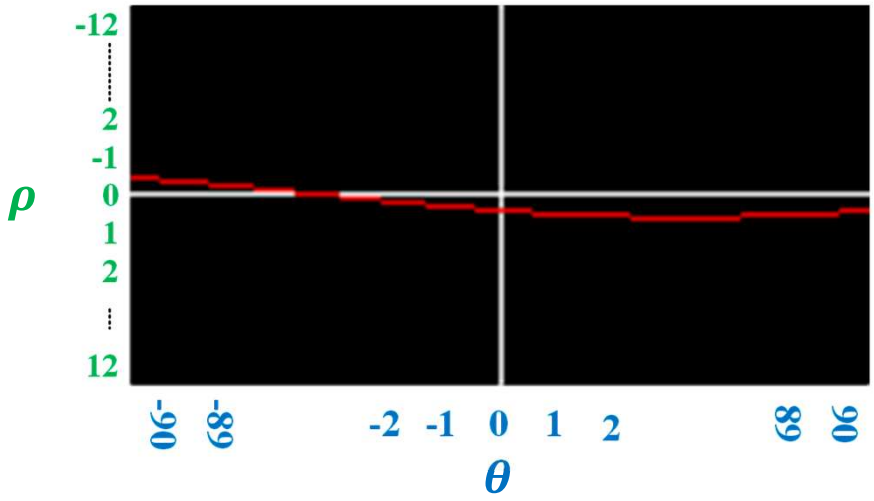
Detection Algorithm

- Parameter Voting

$1x \times \cos\theta_i + 1y \times \sin\theta_i = \rho_i$

θ_i	-90°	-89°	·····	0°	·····	89°	90°
ρ_i	-1	-1	·····	1	·····	1	1

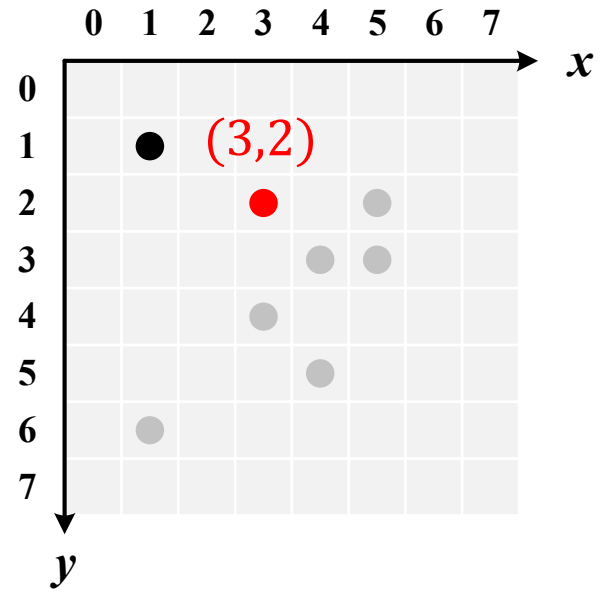




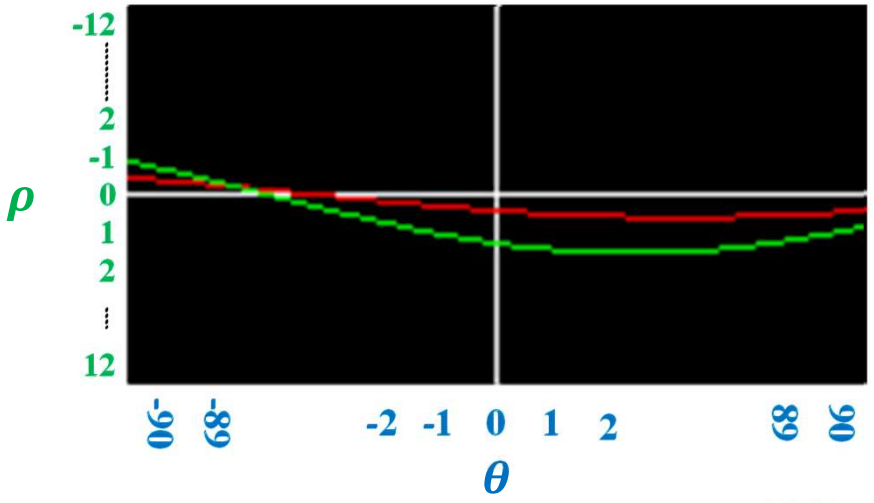
Detection Algorithm

- Parameter Voting

$$\mathbf{3} \times \cos\theta_i + \mathbf{2} \times \sin\theta_i = \rho_i$$



θ_i	-90°	-89°	·····	0°	·····	89°	90°
ρ_i	-2	-2	·····	3	·····	2	2

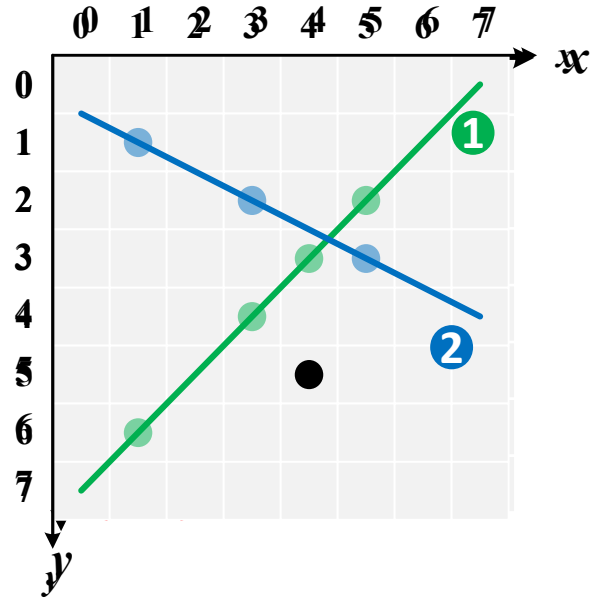


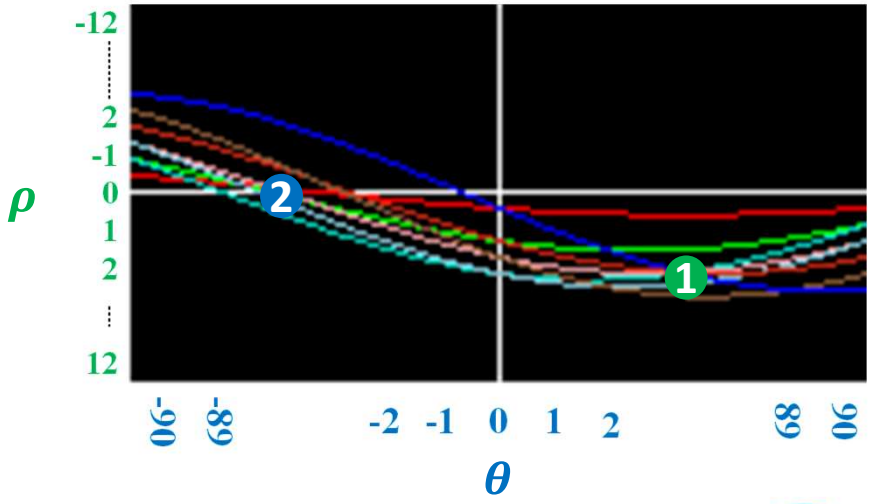
Detection Algorithm

- Parameter Voting

$\mathbf{1} \times \cos\theta_i + \mathbf{6} \times \sin\theta_i = \rho_i$

θ_i	-90°	-89°	·····	0°	·····	89°	90°
ρ_i	-6	-6	·····	1	·····	6	6



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